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Solutions to written exam for the M. Sc in Economics
Economics of Exchange Rates

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Number of questions: This exam consists of 2 questions.

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1. **UIP, CPI and Carry Trade**

(a) Consider the CIP relation

$$\left(1+R_t^*\right)=\frac{1}{S_t}\left(1+R_t\right)F_t \tag{1}$$

and the UIP relation (assuming no risk premium)

$$\left(1+R_t^*\right)=\left(1+R_t\right)\left[E_t\left(\frac{S_{t+1}}{S_t}\right)\right] \tag{2}$$

where notation is standard. These two relations are often motivated using arbitrage arguments. Explain how these two relations are derived using arbitrage arguments.

Answer: **CIP** is an arbitrage relationship between interest rate differentials and forward and spot exchange rates. If domestic and foreign assets are identical and there are no transaction costs or barriers to arbitrage across countries, then arbitrage ensures that any interest rate differential is reflected in movements of exchange rates at the same maturity. In the CIP relation stated above, s_t is the spot exchange rate, f_t^1 is the forward exchange rate at time t for delivery at time $t+1$, i_t is the domestic interest rate and i_t^* is the foreign interest rate.

UIP states that if agents are risk–neutral and have rational expectations, then the return from holding one currency instead of another should be offset by the opportunity cost of holding bonds in a certain currency, i.e., the expected change in spot exchange rates must be equal to the interest differential.

(b) Another approach is to derive these relations using a small-open economy model. Assume that this model is populated by a representative agent that maximizes expected lifetime utility

$$U=\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t u\left(C_t,M_t/P_t\right) \tag{3}$$

subject to the budget constraint measured in foreign currency units

$$S_tB_{t+1}+B_{t+1}^*=S_tB_t\left(1+R_{t-1}\right)+B_t^*\left(1+R_{t-1}^*\right) \tag{4}$$

$$+S_t\left(M_t - M_{t+1}\right) + x_{t-1}\left(F_{t-1} - S_t\right) + S_t P_t\left(Y_t - C_t\right)$$

where notation is standard. Derive the first order conditions and show that these imply both the CIP relation in equation (1) and the UIP relation in equation (2).

Answer: In order to derive the first order conditions we define the Lagrangian in the following way

$$\mathcal{L} = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t u\left(C_t, M_t/P_t\right) + \mathbb{E}_t \sum_{t=0}^{\infty} \lambda_t \left[S_t B_{t+1} + B^*_{t+1} - S_t\left(M_t - M_{t+1}\right)\right.$$

$$\left.-S_t B_t\left(1 + R_{t-1}\right) - B^*_t\left(1 + R^*_{t-1}\right) + S_t P_t\left(Y_t - C_t\right) + x_{t-1}\left(F_{t-1} - S_t\right)\right]$$

and derive the FOC's

$$\frac{\partial \mathcal{L}}{\partial x_t} = \mathbb{E}_t\left[\lambda_{t+1}\left(F_t - S_{t+1}\right)\right] = 0 \tag{5}$$

$$\frac{\partial \mathcal{L}}{\partial B_{t+1}} = \mathbb{E}_t\left[\lambda_t S_t + \lambda_{t+1} S_{t+1}\left(1 + R_t\right)\right] = 0 \tag{6}$$

$$\frac{\partial \mathcal{L}}{\partial B^*_{t+1}} = \mathbb{E}_t\left[\lambda_t + \lambda_{t+1}\left(1 + R^*_{t+1}\right)\right] = 0 \tag{7}$$

To derive CIP we divide (5) by (6) to get

$$\frac{\mathbb{E}_t\left[\lambda_{t+1}\right] F_t}{\lambda_t S_t} = \frac{1}{1 + R_t}$$

and using (7) we find the CIP relation

$$\frac{F_t}{S_t} = \frac{1}{1 + R_t}$$

To derive UIP we make use of the fact that the covariance between λ_{t+1} and S_{t+1} that we have have in equations (5) and (6) can be written as

$$\operatorname{cov}\left(\lambda_{t+1}, S_{t+1}\right) = \mathbb{E}_t\left[\lambda_{t+1} S_{t+1}\right] - \mathbb{E}_t\left[\lambda_{t+1}\right] \mathbb{E}_t\left[S_{t+1}\right]$$

Divide by S_t which is known at time t such that

$$\operatorname{cov}\left(\lambda_{t+1}, S_{t+1}/S_t\right) = \mathbb{E}_t\left[\lambda_{t+1} S_{t+1}/S_t\right] - \mathbb{E}_t\left[\lambda_{t+1}\right] \mathbb{E}_t\left[S_{t+1}/S_t\right]$$

Using (6) we know that $\mathbb{E}_t\left[\lambda_{t+1} S_{t+1/S_t}\right] = \frac{\lambda_t}{1+R_t}$ and from (7) we have that $\lambda_t = \mathbb{E}_t\left[\lambda_{t+1}\right]\left(1 + R^*_t\right)$. Insert these above and rearrange such that

$$\mathbb{E}_t\left[\frac{S_{t+1}}{S_t}\right] = \frac{1 + R^*_t}{1 + R_t} - \frac{\operatorname{cov}\left(\lambda_{t+1}, \frac{S_{t+1}}{S_t}\right)}{\mathbb{E}_t\left[\lambda_{t+1}\right]}$$

where the last part on the LHS is the risk premium. If we assume that the covariance term is equal to zero, the risk premium is zero and we arrive at the UIP relations.

(c) Summarize the empirical evidence on the CIP and the UIP relations above.

Answer: **CIP:** The main result from the literature is that there are very few profitable trading opportunities, in pther words, CIP tends to hold on average. At the same time we often observe a maturity effect such that the existence of profitable opportunities is an increasing function of the length of the period to maturity of the underlying financial instruments. An alternative way of testing CIP is to examine whether news announcements affect exchange rates or the CIP relation. Using this approach the standard result is that new information immediately is reflected in the prices to eliminate any arbitrage opportunity. The overall conclusion from the empirical literature is that we cannot reject CIP.

UIP: In the empirical literature we often reject this hypothesis implying that the foreign exchange market is not efficient. One so called *stylized fact* is that the exchange rate moves in the opposite direction, instead of a depreciated currency if home interest rate is increased (see the UIP relation above), the exchange rate tends to appreciate. This empirical regularity is called forward premium puzzle. One explanation for the failure of UIP is the existence of a risk premium (or the assumption that home and foreign bonds are not identical), agents are not risk neutral and they require a risk premium, a higher rate of return to compensate for the risk of holding foreign currency. The empirical result is that there exists a risk premium and that it is time-varying. Tests of UIP also consider the assumption of rational expectations. Empirical tests suggest that this assumption also is rejected. Overall result is that we reject UIP, there exists a time-varying risk premium and we reject rational expectations.

(d) Explain, in words, how the empirical evidence on UIP can be exploited by formulating carry trade strategies. Summarize the empirical findings related to carry trade. Is it possible to obtain a significant excess return using carry trade?

Answer: The empirical evidence on UIP indicates that low interest currencies tend, on average, to depreciate, not appreciate as UIP states. A trading strategy that borrows in the low interest currency and lends in the high interest currency will, according to the failure of UIP, be profitable on average. This strategy is known as the "carry trade". The empirical literature estimating carry trade profits usually find high Sharpe ratios indicating high profit per unit risk. Sharpe ratios from carry trade often exceed the Sharpe ratio of the S&P 500 index. As expected, the profit is affected when taking bid-ask prices into account. Even though many studies find positive profits, these profits are not economically significant. To generate an annual payoff of 1 million dollars, the agent must bet 28.6 million pounds every month according to Burnside et.al. There are large diversification gains, optimally-weighted portfolios (portfolio that maximizes the Sharpe ratio) generate larger Sharpe ratios. Finally, the literature fails to find significant risk factors correlated to carry trade returns leading to the conclusion that payoffs from carry trade are not compensation for risk.

2. **The Dornbusch overshooting model** Consider the Dornbusch overshooting model where we have added an exogenous risk premium to the UIP relation

$$r - r^* - rp = E\dot{s}^e \tag{1}$$

$$E\dot{s} = \theta\left(\bar{s} - s\right) \tag{2}$$

$$m - p = \eta y - \sigma r \tag{3}$$

$$y^d = \beta + \alpha\left(s - p + p^*\right) + \phi y - \lambda r \tag{4}$$

$$\dot{p} = \pi\left(y^d - y\right). \tag{5}$$

(a) Explain the main assumptions and economic mechanisms underlying the Dornbusch model including an interpretation of the equations above.

Answer: Equation (1) is the UIP relation where we have assumed that domestic and foreign bonds are not perfect substitutes and therefore added a risk premium which is assumed to be constant. Equation (2) describes the expectations formation where θ is the speed of adjustment, $\bar{s}$ is the long–run equilibrium exchange rate and s is the current exchange rate. The equation states that the expected rate of depreciation is proportional to the deviation from the long–run equilibrium. If s is above $\bar{s}$, then the exchange rate is expected to appreciate. Equation (3) is a standard money demand function where real balance is a function of output and the interest rate. Equation (4) is a standard aggregate demand function where demand is given by output, interest rate and the real exchange rate (representing foreign demand for domestic goods). Finally, equation (5) states that the rate of inflation is determined by the gap between aggregate demand and aggregate supply. This model describes a small open economy, i.e., an economy facing a fixed world interest rate. We also assume that output is fixed and prices are sticky. PPP holds only in the long–run.

(b) Derive the money market and goods market equilibrium curves and illustrate the model in a graph. Explain how the slopes of the money market and goods market equilibrium curves are affected by the inclusion of an exogenous risk premium.

Answer: Consider first the money market schedule: Combine equations (1) and (2)

$$r - r^* - rp = \theta(\bar{s} - s)$$

and insert the solution of r from the money demand function (3) such that we find

$$s = \bar{s} - \frac{1}{\theta}\left[\frac{p - m + \eta y}{\sigma} - r^* - rp\right]$$

and then we solve for p

$$p = -\sigma\theta\left(\bar{s} - s\right) + m - \eta y + \sigma r^* + \sigma rp$$

and the slope is

$$\frac{dp}{dp} = -\sigma\theta.$$

In order to derive the goods market equilibrium schedule we first insert the expression for aggregate demand, equation (4), into the the expression for inflation, equation (5) and use the money demand function in (3) such that

$$\dot{p} = \pi \left[\beta + \alpha \left(s - p + p^* \right) + \left(\phi - 1 \right) y - \frac{\lambda}{\sigma} \left(p - m + \eta y \right) \right].$$

In equilibrium, $\dot{p} = 0$ giving us the following solution for the price level

$$p = \frac{\alpha}{\alpha + \frac{\lambda}{\sigma}} s + \beta + \alpha p^* + \frac{\lambda}{\sigma} m + \left(\phi - 1 - \frac{\lambda\eta}{\sigma} \right) y$$

such that the slope is

$$1 > \frac{\alpha}{\alpha + \frac{\lambda}{\sigma}} > 0.$$

As can be seen above when determining the slopes of the money market and goods market equilibrium curves, the risk premium does not affect the slopes. The relative position of the curves, on the other hand, will be affected since the risk premium is assumed to be nonzero.

(c) Derive the overshooting effect, i.e., derive an expression for $\frac{ds}{dm}$. What determines the extent of overshooting?

Answer: Use UIP to solve for r, insert into the money demand function, use equation (3) and rearrange such that

$$p - m = -\eta y + \sigma r^* + \sigma rp + \sigma\theta \left(\bar{s} - s \right).$$

Take the total differential of this equation such that

$$\mathrm{d}p - \mathrm{d}m = -\eta \mathrm{d}y + \sigma \mathrm{d}r^* + \sigma \mathrm{d}rp + \sigma\theta \left(\mathrm{d}\bar{s} - \mathrm{d}s \right)$$

and by noting that prices are sticky in the short–run $\mathrm{d}p = 0$, long–run homogeneity ensures that $\mathrm{d}\bar{s} = \mathrm{d}m$, and that y, r^* and rp are constant we obtain

$$-\mathrm{d}m = \sigma\theta \left(\mathrm{d}m - \mathrm{d}s \right)$$

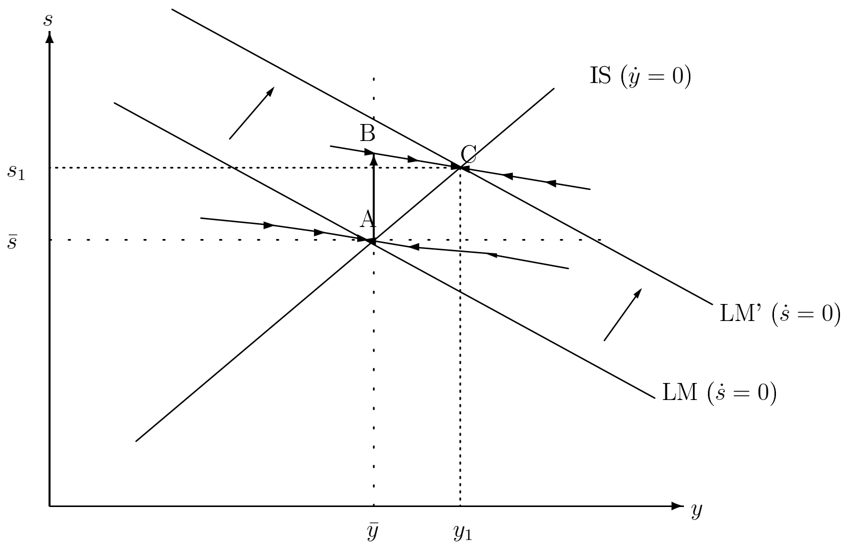
implying that

$$\frac{\mathrm{d}s}{\mathrm{d}m} = 1 + \frac{1}{\sigma\theta}.$$

The risk premium does not affect the size of the overshooting effect.

(d) Consider the Mundell-Fleming model. Illustrate the model in the $y - s$ plane and show how expansionary monetary policy affects the exchange rate and output. Compare the results to the effects of expansionary monetary policy in the Dornbusch model.

Answer: The model is illustrated in the graph below. Expansionary monetary policy in the perfect–foresight Mundell–Fleming model. An increase in m will lead to a shift in the LM–curve up to the right. For a given y, higher m implies a higher s (a depreciated exchange rate) which can be seen from the LM–equation. Since the economy is always on a saddlepath, the exchange rate jumps up to the new saddlepath, the exchange rate jumps from point A to point B. At point B, the economy is on a new saddlepath. What happens next? Foreign demand will increase leading to higher output. Output increases and the economy moves along the saddlepath towards the new long–run equilibrium at point C. Total effect: Depreciated currency (an overshooting effect) and higher output (prices are fixed).



(e) What are the main differences between the Dornbusch model and the Mundell-Fleming model?

Answer: There are two main differences between this model and the M-F model above. In the Dornbusch model output is fixed but the price level is not. In the M-F model it is the opposite. Prices are sticky in the Dornbusch model but the asset market (the exchange rate market) will react immediately to monetary policy. There is overshooting in both models but the mechanisms are different.